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A Monte Carlo Study on Multiple Output Stochastic Frontiers: A Comparison of Two Approaches

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Abstract

In the estimation of multiple output technologies in a primal approach, the main question is how to handle the multiple outputs. Often an output distance function is used, where the classical approach is to exploit its homogeneity property by selecting one output quantity as the dependent variable, dividing all other output quantities by the selected output quantity, and using these ratios as regressors (OD). Another approach is the stochastic ray production frontier (SR) which transforms the output quantities into their Euclidean distance as the dependent variable and their polar coordinates as directional components as regressors. A number of studies have compared these specifications using real world data and have found significant differences in the inefficiency estimates. However, in order to get to the bottom of these differences, we apply a Monte-Carlo simulation. We test the robustness of both specifications for the case of a Translog output distance function with respect to different common statistical problems as well as problems arising as a consequence of zero values in the output quantities.

Although, our results partly show clear reactions to statistical misspecifications, on average none of the approaches is superior. However, considerable differences are found between the estimates at single replications. In the case of zero values in the output quantities, the SR clearly outperforms the OD, although this advantage nearly vanishes when zeros are replaced by a small number.

Keywords: Multiple Outputs, SFA, Monte Carlo Simulation, Stochastic Ray Production Frontier, Output Distance Function

JEL Classification: C21 · C40 · D24

1 Introduction

Input and output distance functions and their parametric estimation form, foremost the stochastic frontier function (SFA), are widely applied instruments to measure productivity when technical inefficiency is taken into account. In the case of multiple outputs—given that the underlying production technologies differ significantly—it is common to use a dual approach and to estimate either a cost function, a profit function, or a system of shadow price equations. In cases where standard economic assumptions such as cost minimisation or profit maximisation do not hold, e.g. in some public sector services, or if price data are not available or unvarying, a primal approach to estimate multiple output production functions is an attractive option.

The difficulty with the estimation of output distance functions for multiple outputs is that there is no natural choice of a dependent variable (Kumbhakar and Lovell, 2000). Therefore, it is common to select one output quantity, say y_M, as the dependent variable and to use the normalised other output quantities y_m/y_M as explanatory variables (in addition to the inputs). Alternatively, Kumbhakar and Lovell (2000) suggested using the output norm $||y||$ as the dependent variable and the correspondingly normalised output quantities $y_m/||y||$ as explanatory variables. Taking the latter approach into consideration, Löthgren (1997, 2000) introduced a further somewhat different concept to handle multiple outputs in the same framework by introducing the multiple-output stochastic ray frontier production function. In this specification, the simple ratios $y_m/||y||$ are replaced by polar coordinates, i.e. replacing the Cartesian coordinates by polar coordinates. To the authors' knowledge, these different normalisation approaches have only been compared using real world data (e.g. Whiteman, 1999; Fousekis, 2002; Zhang and Garvey, 2008), and although efficiency estimates showed considerable deviations (e.g. Zhang and Garvey (2008) find mean deviations of up to 22 %), it is still unclear which approach performs better.

In order to get to the bottom of these empirical findings, we compare the performance of the classical normalisation approach with one output with the multiple-output stochastic ray frontier approach by means of a Monte Carlo simulation. We test the reaction of both approaches given several common data problems, e.g. endogeneity of the regressors, heteroscedasticity of the inefficiency term and noise term, or zero values in the output data of some observations.

The article is structured as follows: section two provides a short overview of the two normalisation concepts compared in the Monte Carlo simulation; section three describes the data generating process and the design of the Monte Carlo simulation; section four presents and discusses the results; and finally section five concludes.

2 Estimating multiple output distance functions in a primal approach

Following Kumbhakar and Lovell (2000, chap. 3.2.3.), the output distance function for multiple outputs can be estimated by applying the approach used in Stochastic Frontier Analysis (SFA). Starting from the SFA in the single-output case $y = f(x;\beta) \cdot \exp\{v - u\}$, where y is the output quantity, x is a vector of input quantities, β is a corresponding vector of parameters, and $\exp\{v - u\}$ is an error term decomposed into a noise term v and an inefficiency term u. By exploiting the fact that in the single output case $y/f(x) = \delta(x, y)$, with $\delta(x, y)$ the Shepardian distance function, one can rewrite the stochastic frontier model for the multiple-output case as

$$1 = \delta(x, y) \cdot \exp\{u - v\} \tag{1}$$

As $\delta(x,y) \leq 1$, $\exp\{u-v\} \geq 1$ can be used as a reciprocal measure of technical efficiency. There are now two possibilities to convert equation (1) into an estimable regression model:

1. By utilising the property of homogeneity of degree one in outputs $\delta(x, \lambda y) = \lambda\delta(x,y)$ $\forall\, \lambda > 0$ and setting λ to y_M^{-1} (e.g. Coelli and Perelman, 1996; Fuentes et al, 2001), one yields $\delta(x, y/y_M) = y_M^{-1} \cdot \delta(x,y)$ which leads to $\delta(x,y) = y_M \cdot \delta(x, y/y_M)$. By inserting the last equation into equation (1) and dividing by y_M the final estimation equation is denoted by

$$y_M^{-1} = \delta\left(x, \frac{y}{y_M}\right) \cdot \exp\{u - v\} \tag{2}$$

2. Another alternative is the stochastic ray production frontier developed by Löthgren (2000). Multiple outputs are modelled by decomposing the vector of M output quantities $y = ||y|| \cdot p(\vartheta)$ into a scalar distance component, the Euclidean distance $||y|| = \left(\sum_{m=1}^{M} y_m^2\right)^{1/2}$, and a vector of directional measures $p(\vartheta)$ with $\vartheta = (\vartheta_1, \ldots, \vartheta_{M-1})$ a vector of polar coordinates where $\vartheta_m \in [0, \pi/2]^{M-1}$ $\forall\; m = 1, \ldots, M-1$ and $\sin(\vartheta_0) = \cos(\vartheta_M) = 1$, and with $p : [0, \pi/2]^{M-1} \to [0,1]^M$ a function which transforms the polar-coordinate angle vector ϑ to the output-mix vector $p(\vartheta) = y/||y||$, with norm $||p(\vartheta)|| = 1$. The directional vector is measured as

$$p_m(\vartheta) = \frac{y_m}{||y||} = \cos(\vartheta_m) \prod_{j=0}^{M-1} \sin(\vartheta_j) \quad \forall\, m = 1, \ldots, M$$

and ϑ is recursively defined by

$$\vartheta_m(y) = \arccos\left(y_m / \left[||y|| \prod_{j=0}^{M-1} \sin \vartheta_j\right]\right) \quad \forall\, m = 1, \ldots, M.$$

The output distance function can then be expressed as

$$\omega(x,y) = ||y|| / ||f(x,\vartheta)p(\vartheta)|| = ||y|| / f(x,\vartheta)$$

with $f(x,\vartheta) = \sup(||y|| \mid ||y|| \cdot p(\vartheta) \in P(x))$, with $P(x)$ the output set defined by the technology $P : x \to y$. Inserting the upper definition into equation (1) yields

$$||y|| = f\left(x, \vartheta(y)\right) \cdot \exp\{v - u\} \tag{3}$$

Expressing the output distance functions in (2), and (3) in parametric form by applying the flexible Translog form for $n = 1, \ldots, N$ inputs and $m = 1, \ldots, M$ outputs gives the following specifications

$$\begin{aligned} -\ln(y_M) =& \alpha_0 + \sum_{m=1}^{M-1} \alpha_m \ln(y_m/y_M) + 0.5 \sum_{m=1}^{M-1} \sum_{j=1}^{M-1} \alpha_{mj} \ln(y_m/y_M) \ln(y_j/y_M) \\ &+ \sum_n \beta_n \ln(x_n) + 0.5 \sum_n \sum_l \beta_{nl} \ln(x_n) \ln(x_l) \\ &+ \sum_{m=1}^{M-1} \sum_n \gamma_{mn} \ln(y_m/y_M) \ln(x_n) + u - v \end{aligned} \tag{4}$$

and

$$\ln(||y||) = \alpha_0 + \sum_{m=1}^{M-1} \alpha_m \vartheta_m + 0.5 \sum_{m=1}^{M-1} \sum_{j=1}^{M-1} \alpha_{mj} \vartheta_m \vartheta_j + \sum_n \beta_n \ln(x_n) \tag{5}$$

$$+ 0.5 \sum_n \sum_l \beta_{nl} \ln(x_n) \ln(x_l) + \sum_{m=1}^{M-1} \sum_n \gamma_{mn} \vartheta_m \ln(x_n) - u + v,$$

where α_0 is a scalar intercept, $\alpha = [\alpha_m]$, $m = 1, \ldots, M-1$ and $\beta = [\beta_n]$, $n = 1, \ldots, N$ are vectors of first-order parameters and $A = [\alpha_{mj}]$, $m, j = 1, \ldots, M-1$, $B = [\beta_{nl}]$, $n, l = 1, \ldots, N$, and $C = [\gamma_{mn}]$, $m = 1, \ldots, M-1$, $n = 1, \ldots, N$ are matrices of second-order parameters, where A and B are symmetric, i.e. $\alpha_{mj} = \alpha_{jm}\ \forall\ m, j = 1, \ldots, M-1$ and $\beta_{nl} = \beta_{ln}\ \forall\ n, l = 1, \ldots, N$. As in the usual single-output stochastic production frontier, it is assumed that the noise term v follows a normal distribution with mean 0 and variance σ_v^2 and the inefficiency term u follows a positively truncated normal distribution (unless other distributional forms are assumed).

3 Data generating process and design of the Monte Carlo simulation

It is apparent that the functional forms defined in equation (4) and (5) are not nested into each other. By choosing either equation (4) or (5) as the data generating process (DGP) one would discriminate against the other functional form. Therefore, following Färe et al (2010) both functional forms, (4) and (5), are chosen for the DGP to test the robustness of either specification.

We run the simulation with a simple setting of two input variables x_n with $n = 1, 2$, and two output variables y_m with $m = 1, 2$. We follow the procedure suggested by Perelman and Santin (2009) to generate the data for the Monte Carlo (MC) simulation. In the first step, the parameters of functions (4) and (5) are chosen. The parameters of function (4) are taken from Perelman and Santin (2009) as a base-line scenario. These parameters—given that the explanatory variables are within specific ranges—fulfil the regularity conditions of the output distance function, i.e. homogeneous of degree one in outputs; convex and continuous in outputs and quasi-convex in inputs; and non-decreasing in y and non-increasing in x. With this set of parameters, the technology exhibits increasing returns to scale (IRS), whilst input-output separability (IOS) is not fulfilled. Further sets of parameters are chosen so that input-output separability (IOS) is fulfilled, i.e. $\gamma_{mn} = 0\ \forall\ m = 1, \ldots M-1, n = 1, \ldots, N$, and/or the technology exhibits constant returns to scale (CRS), i.e. $\sum_n \beta_n = 1$ and $\sum_n \beta_{nk} = 0\ \forall\ k = 1, \ldots, N$. The parameters of these specifications as well as the parameters of the Translog ray production frontier (5) are chosen in a way that the levels and—as far as possible—the first and second derivatives are equal at the sample mean among all 8 specifications. The eight specifications regarding the parameters are summarised in Table 1.

Secondly, following Perelman and Santin (2009), the input quantities are sampled from a uniform distribution over the interval $[5, 50]$. This ensures that the regularity conditions are fulfilled, because they are fulfilled—given the chosen parameters—if the logarithmic input ratios lie in the interval $|\ln x_2 - \ln x_1| \leq 2.5$. The logarithmic output ratios are sampled from a uniform distribution in a way that they lie in the interval $|\ln y_2 - \ln y_1| \leq 1.5$. Thirdly, the inefficiency terms u are sampled from a half-normal distribution $u \sim |N(0, \sigma_u^2)|$. The noise terms v_m are sampled from a normal distribution $v_m \sim N(0, \sigma_v^2)$. Given the generated input quantities and output ratios and the chosen parameters, the

Table 1: Variation of DGP specifications

Nr	Approach	Form
1.1	eq. (2)	Translog (4), CRS, IOS
1.2		Translog (4), CRS, no IOS
1.3		Translog (4), IRS, IOS
1.4		Translog (4), IRS, no IOS
1.5	eq. (3)	Translog (5), CRS, IOS
1.6		Translog (5), CRS, no IOS
1.7		Translog (5), IRS, IOS
1.8		Translog (5), IRS, no IOS

"deterministic" fully efficient output quantities y^* are calculated using equation (4) and equation (5). The sampled output ratios enter ϑ in equation (5) as

$$\vartheta_1 = \arccos\left(\frac{y_1}{\sqrt{y_1^2 + y_2^2}}\right) = \arccos\left(\frac{1}{\sqrt{1 + \exp(2\left(\ln(y_1) - \ln(y_2)\right))}}\right) \tag{6}$$

with $\ln(y_1) - \ln(y_2)$ being the sampled logarithmic output ratios. Finally, the noise term and the inefficiency term are subjoined to the y^* in order to obtain the "observed" output quantities:

$$y_m = y_m^* \exp(v - u). \tag{7}$$

A total population of 2,500 observations is generated based on randomly drawn variables as described above. Then, in each replication of the Monte Carlo simulation, a new sample of 25, 100, or 200 observations is drawn from the population and used for the estimations.

We impose the following specifications on the basic setting to test the robustness of both approaches:[1]

1. Variation of sample size:
   As the quality of the estimates varies with the sample size, we use three different sample sizes: 25, 100, and 200 observations.

2. Different ratios of the standard errors of the error terms:
   σ_u^2 is set to 0.05 and 0.8 so that the average "true" efficiencies are around 86% and 56%, respectively. With $\sigma_v^2 = 0.05$, the ratio σ_u/σ_v is equal to 1 and 4, respectively (Jensen, 2005).

3. Different distributions of the noise term:
   the noise term is simulated either with a normal distribution $v \sim N(0, \sigma_v^2)$ or with a t-distribution $v \sim t(0, \sigma_v^2, 15)$.

4. Correlation of the output ratios with the noise term v and the inefficiency term u:
   A potential problem with the estimation form of the output distance function is that the output ratios appear as regressors in the estimation equation. This could lead

[1] In earlier versions of the analysis we also tested other scenarios, i.e. omitted variables, multicollinearity, different distributions of the inefficiency term, different variances of the noise term. Both specifications performed equally well in all these scenarios.

to inconsistent parameter estimates, as the output ratios might well be endogenous regressors. For instance, this happens when inefficiency and noise affect the different outputs differently (Roibás and Arias, 2004). Inconsistent estimates of the model parameters will in turn have an impact on the estimation of the efficiency term u and might lead to the under or overestimation of the efficiency. As this problem can have different occurrences given the individual observations, this effect will both inflict damage on the ranking as well as the level of the individual efficiencies. To test whether the endogeneity of the output-regressors has an impact on the estimation of the efficiency term, our MC simulation includes scenarios with different noise terms and inefficiency terms for the two output quantities.

5. Impact of returns to scale and input-output separability:
   Following Kumbhakar (2011) non-constant returns to scale and missing input-output separability aggravate endogeneity problems. Therefore, the technology is modelled with constant returns to scale and variable returns to scale, as well as with fulfilled and unfulfilled input-output separability.

6. Heteroscedasticity of the noise term and inefficiency term:
   Following Jensen (2005) we either impose heteroscedasticity of the noise term v^* by $v^* = v \cdot \exp\{\delta_0 + \delta_1 x_1 + \delta_2 x_2\}$, or heteroscedasticity of the inefficiency term u^* by $u^* = u \cdot \exp\{\delta_0 + \delta_1 x_1 + \delta_2 x_2\}$, where δ_0, δ_1, and δ_2 are chosen so that the mean of $\exp\{\delta_0 + \delta_1 x_1 + \delta_2 x_2\}$ is approximately one.

7. Zero output quantities:
   Finally, to test the sensitivity of both specifications towards zeros in the output values, an increasing share of zero valued output observations is introduced into the data, i.e. 0.05, 0.1, 0.3. As it is not possible to generate zero valued outputs with the equation (4), the DGP with zero output quantities will be entirely based on equation (5).

As both approaches use different specifications, a direct comparison of parameter estimates is futile. Therefore, following Coelli and Perelman (2000), the comparison of the quality of the estimates is limited to the efficiency estimates of both approaches. For the comparison between the true and the estimated efficiencies, we apply the following performance measures:

1. Calculate the median absolute deviation (MAD) between the estimated and the true efficiencies for each replication: $\text{median}(|\hat{e}_i - e_i|)$, where $\hat{e}_i$ is the estimated efficiency, e_i is the true efficiency, and subscript i indicates the observation.

2. Following Andor and Hesse (2011), we calculate the mean absolute deviation (MD) between the estimated and the true efficiencies for each replication: $\text{nObs}^{-1} \sum_i |\hat{e}_i - e_i|$, where nObs is the number of observations.

3. As a third criteria, we calculate the average bias of the estimated efficiencies for each replication: $\text{nObs}^{-1} \sum_i (\hat{e}_i - e_i)$.

4. Finally, we calculate the average Spearman rank correlation coefficient (e.g. Gong and Sickles, 1992; Ruggiero, 1999; Jensen, 2005), as the ranking of the efficiencies is often of more interest than the absolute values. Additionally, WAIRDIPs

(Weighted Absolute Inefficiency Rank DIfference Plot) are generated, where the absolute differences in the ranking of the true and estimated efficiencies are averaged over replications and weighted with the sample size (Jensen, 2005).

4 Results

The Monte Carlo experiment was conducted in the statistical programming language "R" (R Development Core Team, 2012) using the add-on package "frontier" (Coelli and Henningsen, 2012) for the stochastic frontier estimations. The simulation includes 594 scenarios, and we conducted 500 replications per scenario.

First we look at the scenarios in which all output quantities are strictly positive so that the standard Translog output distance function (4) can be applied to all the observations without modifications. The median absolute deviations, the mean absolute deviations, the rank correlation coefficients, and the mean biases between the estimated efficiencies and the "true" efficiencies are presented in Table 2. These performance measures are presented as mean values over all 576 scenarios (288,000 replications) and as mean values over scenarios with specific properties of the data generating process—both for the standard Translog output distance function (4), in the following abbreviated as "OD", and the Translog stochastic ray production frontier (5), in the following abbreviated as "SR". Furthermore, Table 2 presents P-values for the tests of significant differences between different properties of the data generating process (obtained by an ANOVA with interaction terms) and P-values for Welch's (1947) t-tests for differences in the average performance of the OD and the SR.[2]

The functional form (OD or SR) that is used in the data generating process has only a very small influence on the precision of the estimated efficiencies. When the output quantities are generated by the OD, the estimated efficiencies are slightly more precise and less biased but the ranking is less precise, regardless of whether the efficiencies are estimated by the OD or the SR. Although non-constant returns to scale and missing input-output separability should theoretically result in inconsistent estimates (Kumbhakar, 2011), we found that these properties of the technology did not have an effect on the precision of the estimated efficiencies.

While heteroscedasticity in the noise term v clearly reduces the precision of the efficiency estimates, heteroscedasticity in the inefficiency term u reduces the median absolute deviation and the bias but increases the mean absolute deviation and reduces the rank correlation coefficient, which indicates that heteroscedasticity in the noise term v is more severe than heteroscedasticity in the inefficiency term u.

As the estimation assumes normally distributed noise terms, generating the noise terms from a t-distribution clearly reduces the precision of the estimates. Similarly, when the noise term and the inefficiency term differ between the two output quantities, the precision of the efficiency estimates deteriorates, as this causes an endogeneity bias (Roibás and Arias, 2004). This endogeneity problem has a very large effect on the (average) bias, where the over-estimation of the efficiencies almost disappears when this problem does not occur.

The variance of the inefficiency term u has a major influence on the performance of the efficiency estimates. While a larger variance of the inefficiency term u and hence, smaller average efficiencies increases the median absolute deviation, the mean absolute deviation

[2] Interaction effects between the different properties of the data generating process are presented in Appendix Tables A1 and A2.

Table 2: Mean performance of the OD and the SR

	MAD OD	MAD SD	P-val	MD OD	MD SR	P-val	RC OD	RC SR	P-val	BIAS OD	BIAS SR	P-val
all scenarios	0.1277	0.1278	0.8454	0.1438	0.1439	0.7087	0.5777	0.5772	0.4142	0.0708	0.0704	0.2757
OD	0.1274	0.1274	0.7710	0.1435	0.1437	0.7395	0.5751	0.5743	0.3607	0.0696	0.0693	0.4942
SR	0.1281	0.1281	0.9902	0.1441	0.1442	0.8436	0.5802	0.5800	0.8072	0.0719	0.0715	0.3916
P-value	0.0080	0.0232		0.0109	0.0177		0.0000	0.0000		0.0000	0.0000	
CRS	0.1277	0.1277	0.9986	0.1438	0.1439	0.8742	0.5777	0.5772	0.5405	0.0707	0.0703	0.3971
IRS	0.1277	0.1278	0.7840	0.1439	0.1440	0.7113	0.5776	0.5772	0.5873	0.0708	0.0705	0.4872
P-value	0.8532	0.5928		0.7749	0.5673		0.9209	0.9601		0.8719	0.7230	
IOS	0.1279	0.1279	0.9595	0.1441	0.1441	0.8715	0.5776	0.5772	0.6586	0.0709	0.0705	0.3595
non-IOS	0.1275	0.1276	0.8220	0.1436	0.1438	0.7138	0.5777	0.5771	0.4761	0.0706	0.0703	0.5321
P-value	0.1261	0.1908		0.0771	0.1354		0.7451	0.7938		0.4293	0.6809	
no heterosced.	0.1260	0.1256	0.4250	0.1399	0.1397	0.6166	0.5951	0.5950	0.8832	0.0772	0.0765	0.1657
heterosc. in u	0.1238	0.1237	0.7538	0.1430	0.1429	0.7234	0.5755	0.5753	0.8463	0.0602	0.0592	0.0893
heterosc. in v	0.1333	0.1339	0.1610	0.1486	0.1492	0.1412	0.5623	0.5613	0.2920	0.0748	0.0755	0.2698
P-value	0.0000	0.0000		0.0000	0.0000		0.0000	0.0000		0.0000	0.0000	
v norm. distr.	0.1265	0.1262	0.3831	0.1422	0.1419	0.4621	0.5843	0.5843	0.9514	0.0681	0.0674	0.1110
v t-distributed	0.1289	0.1293	0.2257	0.1455	0.1459	0.1847	0.5710	0.5700	0.2216	0.0734	0.0735	0.9424
P-value	0.0000	0.0000		0.0000	0.0000		0.0000	0.0000		0.0000	0.0000	
u & v the same	0.1187	0.1187	0.8471	0.1388	0.1389	0.7274	0.5946	0.5946	0.9848	0.0303	0.0299	0.3830
u & v different	0.1367	0.1368	0.9268	0.1488	0.1489	0.8456	0.5607	0.5598	0.2540	0.1113	0.1110	0.4522
P-value	0.0000	0.0000		0.0000	0.0000		0.0000	0.0000		0.0000	0.0000	
$\sigma_u^2 = 0.05$	0.0983	0.0985	0.1932	0.1143	0.1146	0.0396	0.3999	0.4001	0.6566	0.0366	0.0367	0.7450
$\sigma_u^2 = 0.8$	0.1571	0.1570	0.8437	0.1734	0.1733	0.7688	0.7554	0.7543	0.0150	0.1049	0.1041	0.1189
P-value	0.0000	0.0000		0.0000	0.0000		0.0000	0.0000		0.0000	0.0000	
nObs = 25	0.1809	0.1810	0.9206	0.1979	0.1979	0.9742	0.4840	0.4822	0.0580	0.1328	0.1315	0.0773
nObs = 100	0.1074	0.1074	0.7862	0.1241	0.1243	0.5094	0.6140	0.6139	0.9141	0.0419	0.0419	0.8764
nObs = 200	0.0948	0.0948	0.9324	0.1095	0.1096	0.6044	0.6349	0.6355	0.5738	0.0375	0.0378	0.4263
P-value	0.0000	0.0000		0.0000	0.0000		0.0000	0.0000		0.0000	0.0000	

and the bias of the estimated efficiencies, it considerably improves the estimated ranking of the efficiencies. However, a larger variance of the inefficiency term u not only improves the ranking efficiencies but also the *relative* precision of the estimated *inefficiencies*, measured by dividing the median absolute deviation, the mean absolute deviation and the bias by the average inefficiency in each replication.

Finally, the number of observations clearly has a significant influence on the precision of the efficiency estimates, where 25 observations are apparently insufficient to obtain reasonably precise efficiency estimates.

While the average precision of the efficiency estimates clearly depends on the properties of the DGP, it is virtually unaffected by the choice between the OD and the SR. Furthermore, the OD and the SR react in the same way to the modifications of the DGP. However, our results do not contradict previous empirical studies that found considerable differences in the efficiency estimates between the OD and SR (e.g. Whiteman, 1999; Fousekis, 2002; Zhang and Garvey, 2008), because our results reported above only indicate that *on average* the *precision* of the efficiency estimates does not differ between the OD and the SR. This is illustrated for a typical scenario in Figure 1. In some replications of our Monte Carlo simulation, the precision of the OD and the SR is similar (circles close to the horizontal line at MAD OD - MAD SR = 0), but the efficiency estimates considerably differ between the OD and the SR (MAD OD SR $\gg 0$). In other replications, the OD clearly outperforms the SR (MAD OD - MAD SR < 0), while in further replications it is the other way round (MAD OD - MAD SR > 0) so that the average performance is the same for the OD and the SR (mean of MAD OD - MAD SR = 0). The average differences of the efficiency estimates between the OD and the SR (measured in terms of the median absolute deviation, the mean absolute deviation, and the rank correlation between the efficiency estimates of the OD and the SR) are particularly significant when the number of observations is small, the variance of the inefficiency term u is large, and the noise term and the inefficiency term differ between the two output quantities. Unfortunately, we did not find any characteristics that could indicate whether the OD or the SR performs better when the true efficiency estimates are unknown.

However, under the presence of zero output values the OD and the SR generally do not perform equally well. As the OD specification cannot model zero output quantities, we estimated the OD only with the observations that have strictly positive output quantities. Additionally, we estimate equation (4) with a data set, where all zero values in the output quantities are replaced by a small decimal (from now on ODz), in our case 0.01[3]. This quite widespread practice of dealing with zero values in a Translog function has been heavily criticised by N'Guessan et al (2006). Tables 3, 4, and 5 report the results for the median absolute deviations, the Spearman rank correlation and the bias, respectively.[4] Taking into account the fact that the OD can only estimate efficiencies of observations that have strictly positive output quantities, we equally omit these observations from the ODz and the SR in the direct comparison with the OD. The full sample is applied to additionally compare the ODz to the SR.

The MAD and the MD of the OD and the ODz significantly increase in tact with the share of zeros in the output values. This stands in stark contrast to the SR which is virtually unaffected by the share of zeros. However, although both OD and ODz are significantly less robust in the presence of zero values, with the ODz having a slight

[3]We also tested the performance by replacing zeros by 0.0001 and 1 but this did not change the essence of the outcome.

[4]More detailed results are available in Appendix Table A3, A4, A5, and A6.

Table 3: Median absolute deviations for scenarios with zero output quantities

	observations with strictly positive output quantities						all observations		
				P-value	P-value	P-value			P-value
	OD	ODz	SR	OD-ODz	OD-SR	ODz-SR	ODz	SR	ODz-SR
all scenarios	0.1192	0.1146	0.1115	0.0002	0.0000	0.0095	0.1153	0.1115	0.0013
share = 0.05	0.1129	0.1132	0.1108	0.8545	0.3129	0.2393	0.1141	0.1115	0.1996
share = 0.1	0.1156	0.1110	0.1106	0.0258	0.0182	0.8690	0.1122	0.1110	0.5546
share = 0.3	0.1290	0.1195	0.1130	0.0000	0.0000	0.0021	0.1197	0.1120	0.0003
P-value	0.0000	0.0000	0.3081				0.0000	0.8379	
share = 0.05, $\sigma_u^2 = 0.05$	0.0985	0.1010	0.0977	0.0486	0.5257	0.0092	0.1009	0.0979	0.0163
share = 0.1, $\sigma_u^2 = 0.05$	0.0979	0.0996	0.0970	0.1606	0.4527	0.0322	0.1000	0.0971	0.0179
share = 0.3, $\sigma_u^2 = 0.05$	0.1032	0.1013	0.0984	0.1548	0.0003	0.0279	0.1012	0.0972	0.0014
share = 0.05, $\sigma_u^2 = 0.8$	0.1272	0.1255	0.1240	0.6470	0.3847	0.6912	0.1273	0.1251	0.5618
share = 0.1, $\sigma_u^2 = 0.8$	0.1333	0.1223	0.1243	0.0050	0.0222	0.6020	0.1244	0.1250	0.8900
share = 0.3, $\sigma_u^2 = 0.8$	0.1547	0.1376	0.1275	0.0001	0.0000	0.0104	0.1382	0.1269	0.0046
P-value	0.0000	0.0001	0.6803				0.0003	0.7416	
share = 0.05, nObs = 25	0.1641	0.1654	0.1610	0.7999	0.5316	0.3881	0.1664	0.1622	0.4069
share = 0.1, nObs = 25	0.1707	0.1582	0.1603	0.0193	0.0534	0.6832	0.1598	0.1613	0.7708
share = 0.3, nObs = 25	0.1953	0.1731	0.1646	0.0001	0.0000	0.1140	0.1751	0.1645	0.0514
share = 0.05, nObs = 100	0.0945	0.0930	0.0928	0.3538	0.3156	0.9015	0.0939	0.0932	0.6647
share = 0.1, nObs = 100	0.0950	0.0938	0.0925	0.3841	0.0839	0.3650	0.0951	0.0927	0.0708
share = 0.3, nObs = 100	0.1049	0.0993	0.0937	0.0009	0.0000	0.0001	0.0987	0.0923	0.0000
share = 0.05, nObs = 200	0.0800	0.0813	0.0787	0.2053	0.1895	0.0111	0.0821	0.0791	0.0039
share = 0.1, nObs = 200	0.0812	0.0809	0.0791	0.7635	0.0327	0.0626	0.0817	0.0792	0.0090
share = 0.3, nObs = 200	0.0867	0.0860	0.0805	0.4668	0.0000	0.0000	0.0852	0.0793	0.0000
P-value	0.0000	0.0886	0.9333				0.0243	0.9015	

Table 4: Mean rank correlation coefficients for scenarios with zero output quantities

	observations with strictly positive output quantities						all observations		
	OD	ODz	SR	P-value OD-ODz	P-value OD-SR	P-value ODz-SR	ODz	SR	P-value ODz-SR
all scenarios	0.5918	0.6098	0.6143	0.0000	0.0000	0.1698	0.6069	0.6163	0.0036
share = 0.05	0.6115	0.6199	0.6203	0.1379	0.1193	0.9463	0.6098	0.6169	0.2037
share = 0.1	0.6002	0.6138	0.6140	0.0190	0.0163	0.9732	0.6025	0.6122	0.0853
share = 0.3	0.5636	0.5956	0.6087	0.0000	0.0000	0.0254	0.6084	0.6197	0.0388
P-value	0.0000	0.0000	0.0003				0.0118	0.0151	
share = 0.05, $\sigma_u^2 = 0.05$	0.4400	0.4404	0.4448	0.9370	0.3138	0.3471	0.4344	0.4424	0.0879
share = 0.1, $\sigma_u^2 = 0.05$	0.4248	0.4278	0.4358	0.5437	0.0255	0.0967	0.4226	0.4379	0.0014
share = 0.3, $\sigma_u^2 = 0.05$	0.4001	0.4278	0.4325	0.0000	0.0000	0.4124	0.4410	0.4466	0.2427
share = 0.05, $\sigma_u^2 = 0.8$	0.7831	0.7994	0.7958	0.0006	0.0080	0.4255	0.7851	0.7914	0.1834
share = 0.1, $\sigma_u^2 = 0.8$	0.7756	0.7998	0.7921	0.0000	0.0007	0.0875	0.7823	0.7865	0.3691
share = 0.3, $\sigma_u^2 = 0.8$	0.7270	0.7635	0.7850	0.0000	0.0000	0.0000	0.7757	0.7928	0.0002
P-value	0.0002	0.0000	0.6368				0.0000	0.8466	
share = 0.05, nObs = 25	0.4990	0.5204	0.5164	0.0231	0.0635	0.6801	0.5019	0.5089	0.4556
share = 0.1, nObs = 25	0.4818	0.5183	0.5096	0.0002	0.0042	0.3725	0.4901	0.5004	0.2807
share = 0.3, nObs = 25	0.4028	0.5015	0.5053	0.0000	0.0000	0.7312	0.5105	0.5159	0.5690
share = 0.05, nObs = 100	0.6550	0.6608	0.6610	0.5175	0.5022	0.9875	0.6518	0.6589	0.4213
share = 0.1, nObs = 100	0.6434	0.6510	0.6526	0.4032	0.3047	0.8613	0.6455	0.6538	0.3474
share = 0.3, nObs = 100	0.6275	0.6344	0.6494	0.4223	0.0136	0.0897	0.6471	0.6596	0.1478
share = 0.05, nObs = 200	0.6806	0.6785	0.6834	0.8186	0.7520	0.5890	0.6756	0.6829	0.4207
share = 0.1, nObs = 200	0.6755	0.6721	0.6798	0.7122	0.6349	0.4031	0.6719	0.6826	0.2386
share = 0.3, nObs = 200	0.6604	0.6509	0.6714	0.2875	0.2223	0.0223	0.6675	0.6836	0.0660
P-value	0.0000	0.6650	0.9491				0.0017	0.2359	

Table 5: Mean biases for scenarios with zero output quantities

	observations with strictly positive output quantities						all observations		
	OD	ODz	SR	P-value OD-ODz	P-value OD-SR	P-value ODz-SR	ODz	SR	P-value ODz-SR
all scenarios	0.0507	0.0368	0.0394	0.0000	0.0000	0.1257	0.0370	0.0410	0.0197
share = 0.05	0.0420	0.0424	0.0393	0.9114	0.3559	0.3072	0.0441	0.0404	0.2200
share = 0.1	0.0436	0.0346	0.0373	0.0025	0.0332	0.3514	0.0367	0.0389	0.4616
share = 0.3	0.0664	0.0334	0.0417	0.0000	0.0000	0.0073	0.0302	0.0437	0.0000
P-value	0.0000	0.0012	0.2247				0.0000	0.1418	
share = 0.05, $\sigma_u^2 = 0.05$	0.0296	0.0356	0.0296	0.0875	0.9974	0.0871	0.0367	0.0304	0.0698
share = 0.1, $\sigma_u^2 = 0.05$	0.0283	0.0302	0.0273	0.5760	0.7783	0.4026	0.0313	0.0282	0.3585
share = 0.3, $\sigma_u^2 = 0.05$	0.0447	0.0224	0.0330	0.0000	0.0007	0.0028	0.0189	0.0324	0.0001
share = 0.05, $\sigma_u^2 = 0.8$	0.0545	0.0491	0.0490	0.2675	0.2500	0.9838	0.0514	0.0504	0.8374
share = 0.1, $\sigma_u^2 = 0.8$	0.0590	0.0390	0.0473	0.0000	0.0152	0.0733	0.0421	0.0496	0.1089
share = 0.3, $\sigma_u^2 = 0.8$	0.0882	0.0445	0.0504	0.0000	0.0000	0.2347	0.0414	0.0551	0.0068
P-value	0.0009	0.0385	0.8757				0.0723	0.8555	
share = 0.05, nObs = 25	0.1076	0.1063	0.1008	0.8415	0.3086	0.4251	0.1103	0.1025	0.2469
share = 0.1, nObs = 25	0.1150	0.0921	0.0965	0.0008	0.0067	0.5053	0.0987	0.0983	0.9604
share = 0.3, nObs = 25	0.1627	0.1075	0.1076	0.0000	0.0000	0.9935	0.1081	0.1100	0.7823
share = 0.05, nObs = 100	0.0111	0.0113	0.0097	0.9602	0.7044	0.6682	0.0128	0.0108	0.5903
share = 0.1, nObs = 100	0.0097	0.0075	0.0089	0.5306	0.8056	0.6982	0.0086	0.0105	0.5857
share = 0.3, nObs = 100	0.0244	-0.0009	0.0113	0.0000	0.0005	0.0007	-0.0052	0.0135	0.0000
share = 0.05, nObs = 200	0.0074	0.0095	0.0074	0.4550	0.9880	0.4608	0.0091	0.0079	0.6886
share = 0.1, nObs = 200	0.0062	0.0043	0.0065	0.4996	0.9031	0.4206	0.0029	0.0077	0.0895
share = 0.3, nObs = 200	0.0122	-0.0063	0.0062	0.0000	0.0398	0.0000	-0.0124	0.0077	0.0000
P-value	0.0000	0.0004	0.4391				0.0004	0.4056	

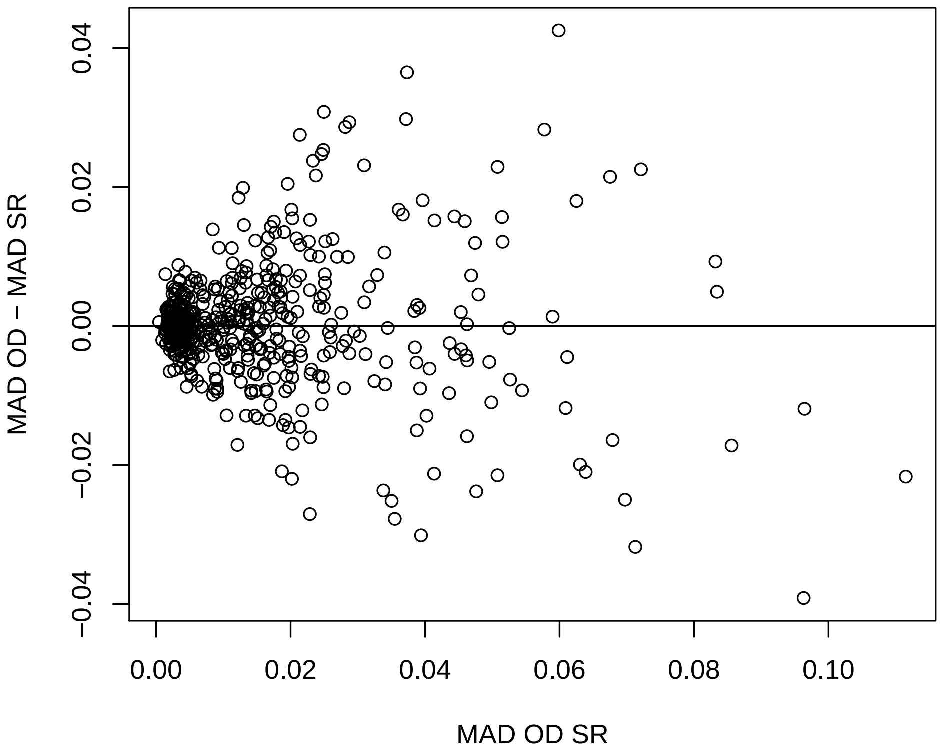

Figure 1: Relationship between the difference of the MAD of the OD and the MAD of the SR (MAD OD - MAD SR) and the MAD between the OD and the SR (MAD OD SR) for the scenario, where the DGP is an OD with CRS and IOS, no heteroscedasticity, the same u and v for both outputs, normally distributed v, $\sigma_u^2 = 0.8$, and 100 observations

advantage over the OD, in absolute terms, the differences are rather small. We find similar results for the influence of the share of zeros on the bias. While the SR is totally unaffected by zero values, the bias of the OD and the ODz significantly increases. All three specifications show a tendency to overestimate the true inefficiency especially when the sample size is small. However, on closer examination, for some scenarios the positive bias of the OD and ODz turns to a distinct negative bias, i.e. a tendency to underestimate the true efficiency. Furthermore, while the positive bias of the SR decreases with increasing sample size, the OD and ODz clearly overshoot the mark for larger samples. For the ODz this applies especially when σ_u^2 is small.

Surprisingly, the ranking correlation of the inefficiencies for all three specifications is significantly affected by the presence of zero values. However, while the ODz and the SR seem to perform equally "bad", the deviation does not lie in the same magnitude as in the case of the OD, as is exemplified in figure 2.

As the OD simply has to cope with fewer observations than the SR under the presence of zeros, the presented results may not come as a surprise. However, the performance of the simple solution used in ODz shows that—despite the differences between ODz and SR being small in absolute terms—the SR should be the first choice given zero output values. How far this result stems from the presence of zeros or is influenced by problems inherent in an inadequate replacement of the zero output values (see the discussion in N'Guessan et al, 2006) remains unsolved.

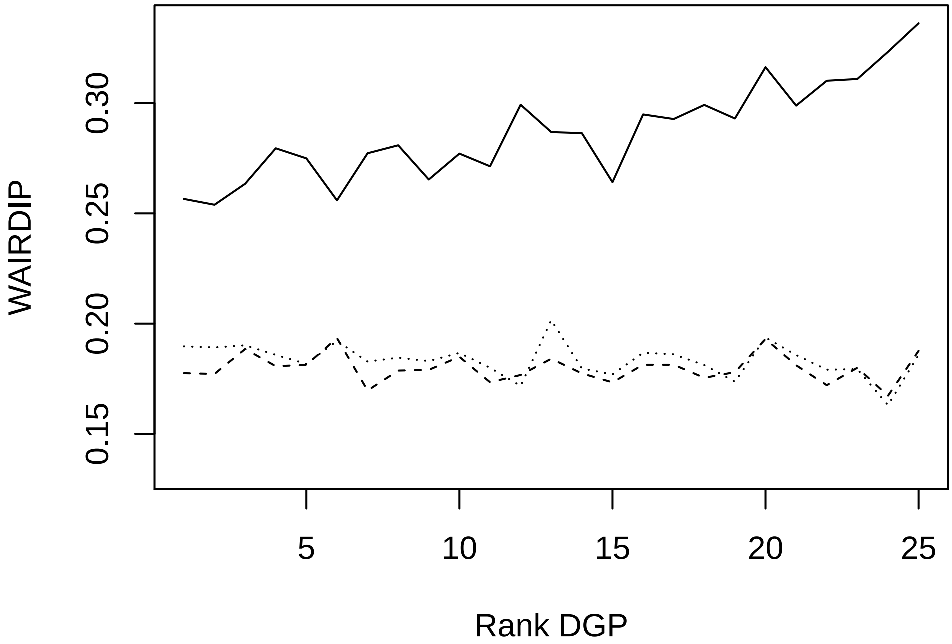

Figure 2: WAIRDIP for the scenario, where the DGP is an SR with 30% of zeros in the output values, normally distributed v, $\sigma_u^2 = 0.8$, and 25 observations; OD (—), ODz($\cdots$), SR (- - -)

5 Conclusion

We compared the standard Translog output distance function (Kumbhakar and Lovell, 2000) with the Translog stochastic Ray production frontier (Löthgren, 2000) by means of a Monte Carlo simulation, exposing both specifications to common data problems. The results indicate that if a considerable share of output quantities is zero, the SR clearly outperforms the OD, as well as the ODz, in which zero values are replaced by a small decimal (in our case 0.01). However, in the case of strictly positive output quantities, on average both specifications perform equally robustly. Nevertheless, our results are in line with earlier empirical findings that show considerable differences between the inefficiency estimates of the OD and the SR at individual estimations. Future research should focus on an indicator to test for the performance of both specifications when the true inefficiencies are unknown.

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Appendix

Table A1: Median absolute deviation and mean absolute deviation of the OD and the SR

	MAD OD	MAD SD	P-val	MD OD	MD SR	P-val
all scenarios	0.1277	0.1278	0.8454	0.1438	0.1439	0.7087
OD	0.1274	0.1274	0.7710	0.1435	0.1437	0.7395
SR	0.1281	0.1281	0.9902	0.1441	0.1442	0.8436
P-value	0.0080	0.0232		0.0109	0.0177	
CRS	0.1277	0.1277	0.9986	0.1438	0.1439	0.8742
IRS	0.1277	0.1278	0.7840	0.1439	0.1440	0.7113
P-value	0.8532	0.5928		0.7749	0.5673	
IOS	0.1279	0.1279	0.9595	0.1441	0.1441	0.8715
non-IOS	0.1275	0.1276	0.8220	0.1436	0.1438	0.7138
P-value	0.1261	0.1908		0.0771	0.1354	
no heterosced.	0.1260	0.1256	0.4250	0.1399	0.1397	0.6166
heterosc. in u	0.1238	0.1237	0.7538	0.1430	0.1429	0.7234
heterosc. in v	0.1333	0.1339	0.1610	0.1486	0.1492	0.1412
P-value	0.0000	0.0000		0.0000	0.0000	
v norm. distr.	0.1265	0.1262	0.3831	0.1422	0.1419	0.4621
v t-distributed	0.1289	0.1293	0.2257	0.1455	0.1459	0.1847
P-value	0.0000	0.0000		0.0000	0.0000	
u & v the same	0.1187	0.1187	0.8471	0.1388	0.1389	0.7274
u & v different	0.1367	0.1368	0.9268	0.1488	0.1489	0.8456
P-value	0.0000	0.0000		0.0000	0.0000	
$\sigma_u^2 = 0.05$	0.0983	0.0985	0.1932	0.1143	0.1146	0.0396
$\sigma_u^2 = 0.8$	0.1571	0.1570	0.8437	0.1734	0.1733	0.7688
P-value	0.0000	0.0000		0.0000	0.0000	
nObs = 25	0.1809	0.1810	0.9206	0.1979	0.1979	0.9742
nObs = 100	0.1074	0.1074	0.7862	0.1241	0.1243	0.5094
nObs = 200	0.0948	0.0948	0.9324	0.1095	0.1096	0.6044
P-value	0.0000	0.0000		0.0000	0.0000	
OD, CRS	0.1273	0.1274	0.9206	0.1435	0.1436	0.8804
SR, CRS	0.1280	0.1280	0.9241	0.1441	0.1441	0.9408
OD, IRS	0.1274	0.1275	0.7550	0.1436	0.1437	0.7491
SR, IRS	0.1281	0.1281	0.9379	0.1442	0.1443	0.8377
P-value	0.9860	0.9605		0.9950	0.9780	
OD, IOS	0.1276	0.1277	0.8629	0.1438	0.1439	0.8558
SR, IOS	0.1282	0.1282	0.9219	0.1443	0.1443	0.9613
OD, non-IOS	0.1271	0.1272	0.8111	0.1433	0.1434	0.7728
SR, non-IOS	0.1279	0.1279	0.9357	0.1440	0.1441	0.8176
P-value	0.8227	0.7427		0.7955	0.7380	
OD, no heterosced.	0.1256	0.1253	0.6755	0.1396	0.1395	0.8302
SR, no heterosced.	0.1263	0.1259	0.4799	0.1402	0.1399	0.6238
OD, heterosc. in u	0.1233	0.1233	0.9729	0.1425	0.1425	0.9874
SR, heterosc. in u	0.1244	0.1241	0.6370	0.1435	0.1432	0.6100
OD, heterosc. in v	0.1332	0.1337	0.3853	0.1485	0.1490	0.4453
SR, heterosc. in v	0.1335	0.1342	0.2659	0.1487	0.1495	0.1895
P-value	0.4140	0.8096		0.4245	0.9340	
OD, v norm. distr.	0.1260	0.1258	0.6569	0.1418	0.1416	0.6741
SR, v norm. distr.	0.1269	0.1265	0.4318	0.1425	0.1422	0.5369
OD, v t-distributed	0.1287	0.1291	0.3694	0.1453	0.1457	0.3506
SR, v t-distributed	0.1292	0.1296	0.4141	0.1458	0.1462	0.3460
P-value	0.4854	0.6718		0.5649	0.7315	
OD, u & v the same	0.1187	0.1187	0.9109	0.1389	0.1389	0.9210
SR, u & v the same	0.1187	0.1187	0.8723	0.1388	0.1390	0.6937

OD, u & v different	0.1360	0.1362	0.7705	0.1482	0.1484	0.7225
SR, u & v different	0.1375	0.1374	0.8775	0.1495	0.1494	0.9430
P-value	0.0048	0.0186		0.0046	0.0316	
OD, $\sigma_u^2 = 0.05$	0.0983	0.0984	0.5800	0.1143	0.1146	0.2335
SR, $\sigma_u^2 = 0.05$	0.0983	0.0985	0.1990	0.1143	0.1146	0.0859
OD, $\sigma_u^2 = 0.8$	0.1564	0.1565	0.8842	0.1728	0.1727	0.9666
SR, $\sigma_u^2 = 0.8$	0.1578	0.1576	0.6755	0.1740	0.1738	0.7115
P-value	0.0063	0.0704		0.0053	0.0299	
OD, nObs = 25	0.1803	0.1803	0.9479	0.1973	0.1973	0.9237
SR, nObs = 25	0.1816	0.1817	0.9398	0.1985	0.1986	0.8892
OD, nObs = 100	0.1072	0.1073	0.6701	0.1240	0.1242	0.4872
SR, nObs = 100	0.1076	0.1075	0.9731	0.1243	0.1243	0.8043
OD, nObs = 200	0.0946	0.0947	0.6931	0.1093	0.1095	0.4569
SR, nObs = 200	0.0950	0.0950	0.7946	0.1096	0.1096	0.9960
P-value	0.2905	0.1733		0.1700	0.0402	
CRS, IOS	0.1279	0.1279	0.9754	0.1440	0.1440	0.9481
IRS, IOS	0.1279	0.1280	0.9182	0.1441	0.1442	0.8699
CRS, non-IOS	0.1275	0.1275	0.9735	0.1436	0.1437	0.8738
IRS, non-IOS	0.1275	0.1277	0.7757	0.1437	0.1438	0.7190
P-value	0.9728	0.9417		0.9817	0.9042	
CRS, no heterosced.	0.1260	0.1256	0.5345	0.1399	0.1397	0.7144
IRS, no heterosced.	0.1260	0.1257	0.6122	0.1399	0.1397	0.7323
CRS, heterosc. in u	0.1238	0.1236	0.7730	0.1430	0.1428	0.7534
IRS, heterosc. in u	0.1239	0.1238	0.8767	0.1430	0.1429	0.8521
CRS, heterosc. in v	0.1333	0.1339	0.3788	0.1486	0.1491	0.3508
IRS, heterosc. in v	0.1334	0.1340	0.2703	0.1487	0.1493	0.2510
P-value	0.9897	0.9807		0.9934	0.9565	
CRS, v norm. distr.	0.1264	0.1261	0.4617	0.1421	0.1418	0.5517
IRS, v norm. distr.	0.1265	0.1262	0.6189	0.1422	0.1420	0.6565
CRS, v t-distributed	0.1289	0.1293	0.4327	0.1455	0.1459	0.3881
IRS, v t-distributed	0.1290	0.1294	0.3531	0.1456	0.1460	0.3113
P-value	0.9479	0.9754		0.9912	0.9835	
CRS, u & v the same	0.1187	0.1187	0.9330	0.1388	0.1389	0.8289
IRS, u & v the same	0.1187	0.1188	0.8504	0.1389	0.1390	0.7820
CRS, u & v different	0.1367	0.1367	0.9434	0.1488	0.1488	0.9807
IRS, u & v different	0.1368	0.1369	0.8407	0.1489	0.1490	0.8016
P-value	0.9372	0.8002		0.9035	0.7632	
CRS, $\sigma_u^2 = 0.05$	0.0982	0.0983	0.6131	0.1142	0.1145	0.2766
IRS, $\sigma_u^2 = 0.05$	0.0983	0.0986	0.1825	0.1144	0.1147	0.0685
CRS, $\sigma_u^2 = 0.8$	0.1571	0.1570	0.8742	0.1734	0.1733	0.8422
IRS, $\sigma_u^2 = 0.8$	0.1571	0.1570	0.9041	0.1734	0.1733	0.8286
P-value	0.7849	0.5926		0.8173	0.5764	
CRS, nObs = 25	0.1809	0.1810	0.9708	0.1979	0.1979	0.9841
IRS, nObs = 25	0.1809	0.1810	0.9169	0.1979	0.1980	0.9794
CRS, nObs = 100	0.1073	0.1074	0.9533	0.1241	0.1242	0.7205
IRS, nObs = 100	0.1074	0.1075	0.7452	0.1242	0.1243	0.5651
CRS, nObs = 200	0.0948	0.0947	0.8352	0.1094	0.1094	0.9362
IRS, nObs = 200	0.0949	0.0949	0.7437	0.1095	0.1097	0.5148
P-value	0.9790	0.9514		0.9889	0.9395	
IOS, no heterosced.	0.1260	0.1257	0.5591	0.1400	0.1398	0.7845
non-IOS, no heterosced.	0.1259	0.1255	0.5864	0.1398	0.1396	0.6640
IOS, heterosc. in u	0.1240	0.1237	0.6810	0.1431	0.1429	0.6132
non-IOS, heterosc. in u	0.1237	0.1236	0.9745	0.1429	0.1429	0.9957
IOS, heterosc. in v	0.1337	0.1344	0.2997	0.1491	0.1496	0.3037
non-IOS, heterosc. in v	0.1330	0.1335	0.3445	0.1482	0.1488	0.2926
P-value	0.6509	0.4707		0.4219	0.3111	
IOS, v norm. distr.	0.1264	0.1261	0.5241	0.1422	0.1419	0.6000
non-IOS, v norm. distr.	0.1265	0.1262	0.5508	0.1422	0.1419	0.6061

IOS, v t-distributed	0.1294	0.1297	0.4548	0.1459	0.1463	0.4294
non-IOS, v t-distributed	0.1285	0.1290	0.3339	0.1451	0.1456	0.2774
P-value	0.0722	0.0997		0.0820	0.1391	
IOS, u & v the same	0.1187	0.1188	0.8672	0.1389	0.1390	0.8057
non-IOS, u & v the same	0.1187	0.1187	0.9160	0.1388	0.1389	0.8049
IOS, u & v different	0.1371	0.1371	0.9377	0.1492	0.1492	0.9973
non-IOS, u & v different	0.1363	0.1365	0.8353	0.1484	0.1486	0.7855
P-value	0.1709	0.2966		0.1029	0.1745	
IOS, $\sigma_u^2 = 0.05$	0.0986	0.0987	0.3822	0.1146	0.1148	0.1400
non-IOS, $\sigma_u^2 = 0.05$	0.0980	0.0982	0.3337	0.1141	0.1143	0.1513
IOS, $\sigma_u^2 = 0.8$	0.1573	0.1571	0.8242	0.1735	0.1734	0.7402
non-IOS, $\sigma_u^2 = 0.8$	0.1570	0.1570	0.9547	0.1732	0.1732	0.9332
P-value	0.6093	0.5020		0.7264	0.5051	
IOS, nObs = 25	0.1811	0.1811	0.9866	0.1981	0.1981	0.9243
non-IOS, nObs = 25	0.1808	0.1809	0.8747	0.1977	0.1978	0.8879
IOS, nObs = 100	0.1076	0.1076	0.8262	0.1243	0.1245	0.5670
non-IOS, nObs = 100	0.1072	0.1072	0.8699	0.1240	0.1241	0.7186
IOS, nObs = 200	0.0951	0.0951	0.9569	0.1097	0.1098	0.8721
non-IOS, nObs = 200	0.0945	0.0946	0.8610	0.1092	0.1094	0.5664
P-value	0.9148	0.8342		0.9512	0.9634	
no heterosced., v norm. distr.	0.1253	0.1247	0.4010	0.1387	0.1383	0.4919
heterosc. in u, v norm. distr.	0.1228	0.1223	0.4315	0.1415	0.1410	0.4111
heterosc. in v, v norm. distr.	0.1314	0.1314	0.9330	0.1463	0.1464	0.8300
no heterosced., v t-distributed	0.1267	0.1265	0.7837	0.1411	0.1411	0.9970
heterosc. in u, v t-distributed	0.1249	0.1251	0.7013	0.1445	0.1447	0.7172
heterosc. in v, v t-distributed	0.1353	0.1365	0.0499	0.1510	0.1520	0.0545
P-value	0.0003	0.0000		0.0004	0.0000	
no heterosced., u & v the same	0.1143	0.1141	0.7387	0.1322	0.1321	0.9115
heterosc. in u, u & v the same	0.1163	0.1165	0.6999	0.1395	0.1397	0.6672
heterosc. in v, u & v the same	0.1254	0.1256	0.7618	0.1449	0.1450	0.7669
no heterosced., u & v different	0.1376	0.1371	0.4370	0.1476	0.1473	0.5710
heterosc. in u, u & v different	0.1313	0.1309	0.4649	0.1465	0.1460	0.4154
heterosc. in v, u & v different	0.1413	0.1423	0.1057	0.1524	0.1534	0.0906
P-value	0.0000	0.0000		0.0000	0.0000	
no heterosced., $\sigma_u^2 = 0.05$	0.0960	0.0960	0.8916	0.1099	0.1100	0.6250
heterosc. in u, $\sigma_u^2 = 0.05$	0.0966	0.0962	0.0728	0.1147	0.1144	0.2728
heterosc. in v, $\sigma_u^2 = 0.05$	0.1022	0.1032	0.0001	0.1183	0.1193	0.0000
no heterosced., $\sigma_u^2 = 0.8$	0.1559	0.1553	0.4006	0.1699	0.1694	0.4620
heterosc. in u, $\sigma_u^2 = 0.8$	0.1510	0.1512	0.8238	0.1713	0.1713	0.9888
heterosc. in v, $\sigma_u^2 = 0.8$	0.1645	0.1647	0.7700	0.1789	0.1791	0.8098
P-value	0.0000	0.0000		0.0000	0.0000	
no heterosced., nObs = 25	0.1807	0.1803	0.6698	0.1951	0.1948	0.8057
heterosc. in u, nObs = 25	0.1767	0.1766	0.8972	0.1974	0.1972	0.7790
heterosc. in v, nObs = 25	0.1854	0.1862	0.4742	0.2013	0.2018	0.5635
no heterosced., nObs = 100	0.1047	0.1046	0.7219	0.1195	0.1195	0.8808
heterosc. in u, nObs = 100	0.1031	0.1030	0.6785	0.1224	0.1223	0.8497
heterosc. in v, nObs = 100	0.1142	0.1147	0.2934	0.1305	0.1311	0.1880
no heterosced., nObs = 200	0.0924	0.0920	0.1589	0.1051	0.1048	0.2666
heterosc. in u, nObs = 200	0.0916	0.0915	0.7028	0.1092	0.1091	0.7316
heterosc. in v, nObs = 200	0.1004	0.1009	0.0829	0.1141	0.1147	0.0313
P-value	0.0000	0.0000		0.0000	0.0000	
v norm. distr., u & v the same	0.1167	0.1167	0.9663	0.1367	0.1367	0.9915
v t-distributed, u & v the same	0.1207	0.1208	0.8173	0.1410	0.1412	0.6289
v norm. distr., u & v different	0.1362	0.1356	0.2394	0.1476	0.1471	0.3380
v t-distributed, u & v different	0.1372	0.1379	0.1502	0.1501	0.1507	0.1767
P-value	0.0000	0.0008		0.0001	0.0447	
v norm. distr., $\sigma_u^2 = 0.05$	0.0941	0.0942	0.7894	0.1101	0.1102	0.5053
v t-distributed, $\sigma_u^2 = 0.05$	0.1024	0.1027	0.1188	0.1185	0.1189	0.0250

v norm. distr., $\sigma_u^2 = 0.8$	0.1588	0.1581	0.2939	0.1742	0.1736	0.2920
v t-distributed, $\sigma_u^2 = 0.8$	0.1555	0.1559	0.4115	0.1726	0.1729	0.4930
P-value	0.0000	0.0000		0.0000	0.0000	
v norm. distr., nObs = 25	0.1827	0.1823	0.5957	0.1990	0.1985	0.5485
v t-distributed, nObs = 25	0.1791	0.1797	0.4841	0.1969	0.1974	0.4988
v norm. distr., nObs = 100	0.1053	0.1052	0.6254	0.1218	0.1217	0.8501
v t-distributed, nObs = 100	0.1094	0.1097	0.3497	0.1265	0.1269	0.2380
v norm. distr., nObs = 200	0.0913	0.0910	0.2251	0.1058	0.1056	0.3637
v t-distributed, nObs = 200	0.0983	0.0986	0.1772	0.1132	0.1136	0.1004
P-value	0.0000	0.0000		0.0000	0.0000	
u & v the same, $\sigma_u^2 = 0.05$	0.1020	0.1020	0.8486	0.1204	0.1205	0.3690
u & v different, $\sigma_u^2 = 0.05$	0.0945	0.0949	0.0709	0.1082	0.1086	0.0486
u & v the same, $\sigma_u^2 = 0.8$	0.1353	0.1355	0.7958	0.1573	0.1573	0.9339
u & v different, $\sigma_u^2 = 0.8$	0.1789	0.1786	0.5965	0.1895	0.1892	0.6281
P-value	0.0000	0.0000		0.0000	0.0000	
u & v the same, nObs = 25	0.1728	0.1731	0.7232	0.1939	0.1942	0.6537
u & v different, nObs = 25	0.1891	0.1889	0.8420	0.2020	0.2017	0.7190
u & v the same, nObs = 100	0.0977	0.0977	0.8145	0.1189	0.1189	0.9427
u & v different, nObs = 100	0.1170	0.1172	0.6803	0.1294	0.1297	0.4810
u & v the same, nObs = 200	0.0854	0.0854	0.6969	0.1038	0.1038	0.8432
u & v different, nObs = 200	0.1042	0.1043	0.7822	0.1152	0.1153	0.4863
P-value	0.0000	0.0000		0.0000	0.0000	
$\sigma_u^2 = 0.05$, nObs = 25	0.1143	0.1143	0.8156	0.1322	0.1325	0.1361
$\sigma_u^2 = 0.8$, nObs = 25	0.2476	0.2477	0.9497	0.2637	0.2634	0.7315
$\sigma_u^2 = 0.05$, nObs = 100	0.0952	0.0954	0.3648	0.1112	0.1115	0.2580
$\sigma_u^2 = 0.8$, nObs = 100	0.1195	0.1194	0.8695	0.1370	0.1371	0.9122
$\sigma_u^2 = 0.05$, nObs = 200	0.0853	0.0856	0.1926	0.0995	0.0998	0.2189
$\sigma_u^2 = 0.8$, nObs = 200	0.1043	0.1041	0.3181	0.1194	0.1193	0.6027
P-value	0.0000	0.0000		0.0000	0.0000	

Table A2: Rank correlation and bias of the OD and the SR

	RC OD	RC SR	P-val	BIAS OD	BIAS SR	P-val
all scenarios	0.5777	0.5772	0.4142	0.0708	0.0704	0.2757
OD	0.5751	0.5743	0.3607	0.0696	0.0693	0.4942
SR	0.5802	0.5800	0.8072	0.0719	0.0715	0.3916
P-value	0.0000	0.0000		0.0000	0.0000	
CRS	0.5777	0.5772	0.5405	0.0707	0.0703	0.3971
IRS	0.5776	0.5772	0.5873	0.0708	0.0705	0.4872
P-value	0.9209	0.9601		0.8719	0.7230	
IOS	0.5776	0.5772	0.6586	0.0709	0.0705	0.3595
non-IOS	0.5777	0.5771	0.4761	0.0706	0.0703	0.5321
P-value	0.7451	0.7938		0.4293	0.6809	
no heterosced.	0.5951	0.5950	0.8832	0.0772	0.0765	0.1657
heterosc. in u	0.5755	0.5753	0.8463	0.0602	0.0592	0.0893
heterosc. in v	0.5623	0.5613	0.2920	0.0748	0.0755	0.2698
P-value	0.0000	0.0000		0.0000	0.0000	
v norm. distr.	0.5843	0.5843	0.9514	0.0681	0.0674	0.1110
v t-distributed	0.5710	0.5700	0.2216	0.0734	0.0735	0.9424
P-value	0.0000	0.0000		0.0000	0.0000	
u & v the same	0.5946	0.5946	0.9848	0.0303	0.0299	0.3830
u & v different	0.5607	0.5598	0.2540	0.1113	0.1110	0.4522
P-value	0.0000	0.0000		0.0000	0.0000	
$\sigma_u^2 = 0.05$	0.3999	0.4001	0.6566	0.0366	0.0367	0.7450
$\sigma_u^2 = 0.8$	0.7554	0.7543	0.0150	0.1049	0.1041	0.1189
P-value	0.0000	0.0000		0.0000	0.0000	
nObs = 25	0.4840	0.4822	0.0580	0.1328	0.1315	0.0773
nObs = 100	0.6140	0.6139	0.9141	0.0419	0.0419	0.8764
nObs = 200	0.6349	0.6355	0.5738	0.0375	0.0378	0.4263
P-value	0.0000	0.0000		0.0000	0.0000	
OD, CRS	0.5751	0.5744	0.5130	0.0696	0.0693	0.5842
SR, CRS	0.5802	0.5800	0.8309	0.0719	0.0714	0.5160
OD, IRS	0.5751	0.5743	0.5231	0.0697	0.0694	0.6749
SR, IRS	0.5802	0.5801	0.8954	0.0719	0.0715	0.5741
P-value	0.9760	0.8953		0.9826	0.9894	
OD, IOS	0.5749	0.5743	0.5898	0.0698	0.0694	0.5697
SR, IOS	0.5803	0.5802	0.9296	0.0720	0.0715	0.4676
OD, non-IOS	0.5753	0.5744	0.4513	0.0695	0.0692	0.6906
SR, non-IOS	0.5802	0.5799	0.7975	0.0717	0.0714	0.6277
P-value	0.5434	0.5936		0.9974	0.9443	
OD, no heterosced.	0.5928	0.5924	0.8051	0.0762	0.0755	0.3869
SR, no heterosced.	0.5975	0.5976	0.9704	0.0783	0.0774	0.2742
OD, heterosc. in u	0.5727	0.5718	0.4962	0.0588	0.0580	0.2918
SR, heterosc. in u	0.5783	0.5788	0.6855	0.0616	0.0605	0.1782
OD, heterosc. in v	0.5598	0.5588	0.5069	0.0739	0.0745	0.4998
SR, heterosc. in v	0.5649	0.5637	0.4091	0.0757	0.0765	0.3765
P-value	0.6263	0.0336		0.5102	0.7263	
OD, v norm. distr.	0.5820	0.5816	0.7062	0.0669	0.0662	0.3227
SR, v norm. distr.	0.5866	0.5871	0.6461	0.0693	0.0685	0.2067
OD, v t-distributed	0.5681	0.5671	0.3580	0.0724	0.0724	0.9721
SR, v t-distributed	0.5739	0.5730	0.4178	0.0745	0.0745	0.9465
P-value	0.1161	0.6621		0.6341	0.8590	
OD, u & v the same	0.5946	0.5946	0.9694	0.0303	0.0299	0.5309
SR, u & v the same	0.5946	0.5946	0.9909	0.0302	0.0298	0.5438
OD, u & v different	0.5555	0.5541	0.2044	0.1090	0.1088	0.7026
SR, u & v different	0.5659	0.5655	0.7221	0.1135	0.1131	0.4977
P-value	0.0000	0.0000		0.0000	0.0000	
OD, $\sigma_u^2 = 0.05$	0.3987	0.3989	0.6920	0.0364	0.0366	0.7118

SR, $\sigma_u^2 = 0.05$	0.4011	0.4012	0.8162	0.0368	0.0368	0.9277
OD, $\sigma_u^2 = 0.8$	0.7515	0.7497	0.0092	0.1029	0.1021	0.2765
SR, $\sigma_u^2 = 0.8$	0.7594	0.7588	0.4052	0.1070	0.1061	0.2639
P-value	0.0000	0.0000		0.0000	0.0000	
OD, nObs = 25	0.4819	0.4793	0.0574	0.1317	0.1303	0.1721
SR, nObs = 25	0.4861	0.4851	0.4317	0.1340	0.1328	0.2567
OD, nObs = 100	0.6112	0.6110	0.9067	0.0409	0.0409	0.9397
SR, nObs = 100	0.6168	0.6168	0.9714	0.0430	0.0428	0.7706
OD, nObs = 200	0.6321	0.6327	0.6907	0.0364	0.0368	0.4116
SR, nObs = 200	0.6377	0.6383	0.6908	0.0387	0.0389	0.7551
P-value	0.2334	0.9835		0.9718	0.7631	
CRS, IOS	0.5776	0.5772	0.7227	0.0709	0.0704	0.5019
IRS, IOS	0.5776	0.5772	0.7871	0.0709	0.0705	0.5324
CRS, non-IOS	0.5777	0.5771	0.6097	0.0706	0.0702	0.5990
IRS, non-IOS	0.5777	0.5771	0.6190	0.0707	0.0704	0.7205
P-value	0.9119	0.9985		0.9959	0.9172	
CRS, no heterosced.	0.5952	0.5950	0.8656	0.0772	0.0765	0.3325
IRS, no heterosced.	0.5951	0.5950	0.9693	0.0773	0.0765	0.3216
CRS, heterosc. in u	0.5755	0.5753	0.8819	0.0602	0.0592	0.2134
IRS, heterosc. in u	0.5755	0.5753	0.9001	0.0602	0.0593	0.2467
CRS, heterosc. in v	0.5623	0.5613	0.4664	0.0748	0.0754	0.4939
IRS, heterosc. in v	0.5624	0.5613	0.4462	0.0749	0.0756	0.3808
P-value	0.9830	0.9935		0.9993	0.9730	
CRS, v norm. distr.	0.5843	0.5843	0.9740	0.0681	0.0673	0.2366
IRS, v norm. distr.	0.5843	0.5843	0.9573	0.0681	0.0674	0.2846
CRS, v t-distributed	0.5710	0.5700	0.3666	0.0734	0.0734	0.9998
IRS, v t-distributed	0.5710	0.5701	0.4090	0.0735	0.0735	0.9187
P-value	0.9796	0.9123		0.9903	0.9998	
CRS, u & v the same	0.5946	0.5946	0.9739	0.0302	0.0298	0.5294
IRS, u & v the same	0.5946	0.5946	0.9954	0.0303	0.0299	0.5453
CRS, u & v different	0.5607	0.5598	0.4029	0.1112	0.1108	0.5243
IRS, u & v different	0.5607	0.5598	0.4373	0.1113	0.1111	0.6698
P-value	0.9976	0.9770		0.9282	0.8094	
CRS, $\sigma_u^2 = 0.05$	0.3999	0.4001	0.7722	0.0365	0.0366	0.9527
IRS, $\sigma_u^2 = 0.05$	0.3999	0.4001	0.7344	0.0366	0.0368	0.6889
CRS, $\sigma_u^2 = 0.8$	0.7554	0.7543	0.0750	0.1050	0.1041	0.2775
IRS, $\sigma_u^2 = 0.8$	0.7554	0.7543	0.0968	0.1049	0.1041	0.2629
P-value	0.9485	0.9976		0.8551	0.6561	
CRS, nObs = 25	0.4841	0.4822	0.1621	0.1328	0.1315	0.2083
IRS, nObs = 25	0.4840	0.4822	0.1995	0.1328	0.1315	0.2151
CRS, nObs = 100	0.6140	0.6139	0.9384	0.0419	0.0418	0.8777
IRS, nObs = 100	0.6140	0.6139	0.9400	0.0420	0.0419	0.9473
CRS, nObs = 200	0.6349	0.6355	0.6931	0.0375	0.0377	0.6698
IRS, nObs = 200	0.6349	0.6355	0.6885	0.0376	0.0380	0.4850
P-value	0.9900	0.9988		0.9918	0.9667	
IOS, no heterosced.	0.5950	0.5948	0.8765	0.0774	0.0766	0.3254
non-IOS, no heterosced.	0.5953	0.5952	0.9581	0.0771	0.0763	0.3286
IOS, heterosc. in u	0.5780	0.5780	0.9513	0.0604	0.0593	0.1733
non-IOS, heterosc. in u	0.5730	0.5725	0.7393	0.0601	0.0592	0.2979
IOS, heterosc. in v	0.5598	0.5589	0.5164	0.0749	0.0755	0.4903
non-IOS, heterosc. in v	0.5648	0.5636	0.3999	0.0747	0.0755	0.3836
P-value	0.0000	0.0000		0.9879	0.9422	
IOS, v norm. distr.	0.5844	0.5845	0.9361	0.0680	0.0672	0.2328
non-IOS, v norm. distr.	0.5842	0.5842	0.9951	0.0682	0.0675	0.2890
IOS, v t-distributed	0.5708	0.5700	0.4784	0.0738	0.0738	0.9273
non-IOS, v t-distributed	0.5713	0.5701	0.3080	0.0730	0.0732	0.8460
P-value	0.3505	0.5695		0.1769	0.2270	
IOS, u & v the same	0.5946	0.5946	0.9968	0.0303	0.0298	0.4617

non-IOS, u & v the same	0.5946	0.5946	0.9818	0.0302	0.0299	0.6188
IOS, u & v different	0.5606	0.5599	0.5324	0.1115	0.1111	0.5291
non-IOS, u & v different	0.5608	0.5597	0.3232	0.1110	0.1108	0.6646
P-value	0.7374	0.8460		0.6385	0.6051	
IOS, $\sigma_u^2 = 0.05$	0.4004	0.4007	0.7060	0.0367	0.0369	0.8155
non-IOS, $\sigma_u^2 = 0.05$	0.3994	0.3995	0.8015	0.0364	0.0365	0.8207
IOS, $\sigma_u^2 = 0.8$	0.7548	0.7538	0.1482	0.1051	0.1041	0.1984
non-IOS, $\sigma_u^2 = 0.8$	0.7561	0.7547	0.0458	0.1048	0.1041	0.3580
P-value	0.0016	0.0055		0.9677	0.6703	
IOS, nObs = 25	0.4836	0.4818	0.1929	0.1328	0.1314	0.1752
non-IOS, nObs = 25	0.4844	0.4825	0.1680	0.1328	0.1316	0.2535
IOS, nObs = 100	0.6141	0.6141	0.9886	0.0421	0.0420	0.8674
non-IOS, nObs = 100	0.6139	0.6137	0.8677	0.0417	0.0417	0.9579
IOS, nObs = 200	0.6351	0.6358	0.6068	0.0378	0.0380	0.6540
non-IOS, nObs = 200	0.6348	0.6352	0.7786	0.0373	0.0376	0.4980
P-value	0.4332	0.3284		0.8600	0.7795	
no heterosced., v norm. distr.	0.6014	0.6016	0.8931	0.0773	0.0762	0.1850
heterosc. in u, v norm. distr.	0.5811	0.5815	0.7752	0.0582	0.0566	0.0624
heterosc. in v, v norm. distr.	0.5703	0.5699	0.7647	0.0689	0.0692	0.7254
no heterosced., v t-distributed	0.5889	0.5884	0.7301	0.0772	0.0767	0.5276
heterosc. in u, v t-distributed	0.5699	0.5691	0.5704	0.0623	0.0619	0.6015
heterosc. in v, v t-distributed	0.5543	0.5526	0.2331	0.0808	0.0818	0.2194
P-value	0.0000	0.0000		0.0000	0.0000	
no heterosced., u & v the same	0.6111	0.6112	0.9208	0.0383	0.0376	0.3638
heterosc. in u, u & v the same	0.5930	0.5934	0.7947	0.0142	0.0135	0.3677
heterosc. in v, u & v the same	0.5796	0.5791	0.7049	0.0383	0.0385	0.8224
no heterosced., u & v different	0.5792	0.5788	0.7597	0.1162	0.1153	0.2423
heterosc. in u, u & v different	0.5579	0.5572	0.5838	0.1063	0.1050	0.0872
heterosc. in v, u & v different	0.5450	0.5434	0.2659	0.1114	0.1125	0.1446
P-value	0.0008	0.0001		0.0000	0.0000	
no heterosced., $\sigma_u^2 = 0.05$	0.4142	0.4148	0.4085	0.0433	0.0428	0.3942
heterosc. in u, $\sigma_u^2 = 0.05$	0.4063	0.4070	0.3572	0.0307	0.0300	0.2241
heterosc. in v, $\sigma_u^2 = 0.05$	0.3792	0.3785	0.3783	0.0357	0.0373	0.0173
no heterosced., $\sigma_u^2 = 0.8$	0.7761	0.7752	0.2273	0.1112	0.1101	0.2425
heterosc. in u, $\sigma_u^2 = 0.8$	0.7447	0.7436	0.1949	0.0897	0.0885	0.1932
heterosc. in v, $\sigma_u^2 = 0.8$	0.7454	0.7440	0.0814	0.1139	0.1137	0.8078
P-value	0.0000	0.0000		0.0000	0.0000	
no heterosced., nObs = 25	0.4986	0.4972	0.4101	0.1371	0.1354	0.1696
heterosc. in u, nObs = 25	0.4835	0.4830	0.7619	0.1282	0.1264	0.1422
heterosc. in v, nObs = 25	0.4700	0.4664	0.0380	0.1332	0.1328	0.8027
no heterosced., nObs = 100	0.6321	0.6323	0.9079	0.0489	0.0485	0.5488
heterosc. in u, nObs = 100	0.6128	0.6124	0.8246	0.0290	0.0282	0.3127
heterosc. in v, nObs = 100	0.5971	0.5970	0.9306	0.0479	0.0489	0.2469
no heterosced., nObs = 200	0.6547	0.6555	0.6422	0.0457	0.0456	0.7952
heterosc. in u, nObs = 200	0.6302	0.6305	0.8659	0.0234	0.0231	0.6105
heterosc. in v, nObs = 200	0.6199	0.6204	0.7353	0.0435	0.0448	0.0464
P-value	0.0000	0.0000		0.0000	0.0000	
v norm. distr., u & v the same	0.6020	0.6018	0.9033	0.0277	0.0274	0.5925
v t-distributed, u & v the same	0.5872	0.5873	0.9268	0.0328	0.0323	0.4857
v norm. distr., u & v different	0.5666	0.5668	0.8393	0.1085	0.1073	0.0635
v t-distributed, u & v different	0.5548	0.5528	0.0627	0.1141	0.1146	0.3610
P-value	0.0001	0.5295		0.5194	0.0019	
v norm. distr., $\sigma_u^2 = 0.05$	0.4053	0.4062	0.1624	0.0295	0.0293	0.7120
v t-distributed, $\sigma_u^2 = 0.05$	0.3945	0.3940	0.4459	0.0437	0.0441	0.4313
v norm. distr., $\sigma_u^2 = 0.8$	0.7632	0.7624	0.2165	0.1067	0.1054	0.0835
v t-distributed, $\sigma_u^2 = 0.8$	0.7476	0.7461	0.0266	0.1032	0.1029	0.6537
P-value	0.0000	0.0000		0.0000	0.0000	
v norm. distr., nObs = 25	0.4900	0.4885	0.2637	0.1362	0.1344	0.0903

v t-distributed, nObs = 25	0.4781	0.4759	0.1184	0.1295	0.1286	0.4242
v norm. distr., nObs = 100	0.6212	0.6217	0.6925	0.0373	0.0370	0.5833
v t-distributed, nObs = 100	0.6068	0.6061	0.5758	0.0465	0.0467	0.7407
v norm. distr., nObs = 200	0.6417	0.6428	0.3937	0.0308	0.0307	0.7648
v t-distributed, nObs = 200	0.6282	0.6281	0.9484	0.0443	0.0450	0.1651
P-value	0.0313	0.0036		0.0000	0.0000	
u & v the same, $\sigma_u^2 = 0.05$	0.4176	0.4175	0.8187	0.0134	0.0132	0.8008
u & v different, $\sigma_u^2 = 0.05$	0.3821	0.3827	0.3846	0.0598	0.0602	0.3853
u & v the same, $\sigma_u^2 = 0.8$	0.7715	0.7717	0.8564	0.0472	0.0465	0.3675
u & v different, $\sigma_u^2 = 0.8$	0.7393	0.7369	0.0002	0.1627	0.1617	0.1188
P-value	0.0000	0.9838		0.0000	0.0000	
u & v the same, nObs = 25	0.4944	0.4943	0.9403	0.1033	0.1021	0.2556
u & v different, nObs = 25	0.4736	0.4701	0.0094	0.1623	0.1610	0.1566
u & v the same, nObs = 100	0.6327	0.6327	0.9579	-0.0035	-0.0036	0.9087
u & v different, nObs = 100	0.5953	0.5951	0.8388	0.0874	0.0873	0.8948
u & v the same, nObs = 200	0.6567	0.6567	0.9923	-0.0090	-0.0090	0.8806
u & v different, nObs = 200	0.6131	0.6142	0.4190	0.0841	0.0846	0.2070
P-value	0.0000	0.0000		0.0000	0.0000	
$\sigma_u^2 = 0.05$, nObs = 25	0.3405	0.3394	0.3325	0.0528	0.0522	0.3241
$\sigma_u^2 = 0.8$, nObs = 25	0.6275	0.6249	0.0030	0.2129	0.2109	0.0758
$\sigma_u^2 = 0.05$, nObs = 100	0.4233	0.4238	0.3033	0.0293	0.0296	0.6242
$\sigma_u^2 = 0.8$, nObs = 100	0.8047	0.8040	0.0297	0.0546	0.0541	0.5001
$\sigma_u^2 = 0.05$, nObs = 200	0.4359	0.4370	0.0046	0.0277	0.0284	0.1876
$\sigma_u^2 = 0.8$, nObs = 200	0.8340	0.8339	0.7070	0.0474	0.0473	0.8127
P-value	0.0000	0.0000		0.0000	0.0000	

Table A3: Median absolute deviations for scenarios with zero output quantities

	observations with strictly positive output quantities						all observations		
				P-value	P-value	P-value			P-value
	OD	ODz	SR	OD-ODz	OD-SR	ODz-SR	ODz	SR	ODz-SR
all scenarios	0.1192	0.1146	0.1115	0.0002	0.0000	0.0095	0.1153	0.1115	0.0013
share = 0.05	0.1129	0.1132	0.1108	0.8545	0.3129	0.2393	0.1141	0.1115	0.1996
share = 0.1	0.1156	0.1110	0.1106	0.0258	0.0182	0.8690	0.1122	0.1110	0.5546
share = 0.3	0.1290	0.1195	0.1130	0.0000	0.0000	0.0021	0.1197	0.1120	0.0003
P-value	0.0000	0.0000	0.3081				0.0000	0.8379	
$\sigma_u^2 = 0.05$	0.0999	0.1007	0.0977	0.2952	0.0030	0.0001	0.1007	0.0974	0.0000
$\sigma_u^2 = 0.8$	0.1384	0.1285	0.1253	0.0000	0.0000	0.1487	0.1300	0.1256	0.0509
P-value	0.0000	0.0000	0.0000				0.0000	0.0000	
nObs = 25	0.1767	0.1656	0.1620	0.0003	0.0000	0.2304	0.1671	0.1627	0.1380
nObs = 100	0.0981	0.0954	0.0930	0.0024	0.0000	0.0058	0.0959	0.0927	0.0001
nObs = 200	0.0826	0.0827	0.0794	0.8771	0.0000	0.0000	0.0830	0.0792	0.0000
P-value	0.0000	0.0000	0.0000				0.0000	0.0000	
share = 0.05, $\sigma_u^2 = 0.05$	0.0985	0.1010	0.0977	0.0486	0.5257	0.0092	0.1009	0.0979	0.0163
share = 0.1, $\sigma_u^2 = 0.05$	0.0979	0.0996	0.0970	0.1606	0.4527	0.0322	0.1000	0.0971	0.0179
share = 0.3, $\sigma_u^2 = 0.05$	0.1032	0.1013	0.0984	0.1548	0.0003	0.0279	0.1012	0.0972	0.0014
share = 0.05, $\sigma_u^2 = 0.8$	0.1272	0.1255	0.1240	0.6470	0.3847	0.6912	0.1273	0.1251	0.5618
share = 0.1, $\sigma_u^2 = 0.8$	0.1333	0.1223	0.1243	0.0050	0.0222	0.6020	0.1244	0.1250	0.8900
share = 0.3, $\sigma_u^2 = 0.8$	0.1547	0.1376	0.1275	0.0001	0.0000	0.0104	0.1382	0.1269	0.0046
P-value	0.0000	0.0001	0.6803				0.0003	0.7416	
share = 0.05, nObs = 25	0.1641	0.1654	0.1610	0.7999	0.5316	0.3881	0.1664	0.1622	0.4069
share = 0.1, nObs = 25	0.1707	0.1582	0.1603	0.0193	0.0534	0.6832	0.1598	0.1613	0.7708
share = 0.3, nObs = 25	0.1953	0.1731	0.1646	0.0001	0.0000	0.1140	0.1751	0.1645	0.0514
share = 0.05, nObs = 100	0.0945	0.0930	0.0928	0.3538	0.3156	0.9015	0.0939	0.0932	0.6647
share = 0.1, nObs = 100	0.0950	0.0938	0.0925	0.3841	0.0839	0.3650	0.0951	0.0927	0.0708
share = 0.3, nObs = 100	0.1049	0.0993	0.0937	0.0009	0.0000	0.0001	0.0987	0.0923	0.0000
share = 0.05, nObs = 200	0.0800	0.0813	0.0787	0.2053	0.1895	0.0111	0.0821	0.0791	0.0039
share = 0.1, nObs = 200	0.0812	0.0809	0.0791	0.7635	0.0327	0.0626	0.0817	0.0792	0.0090
share = 0.3, nObs = 200	0.0867	0.0860	0.0805	0.4668	0.0000	0.0000	0.0852	0.0793	0.0000
P-value	0.0000	0.0886	0.9333				0.0243	0.9015	
$\sigma_u^2 = 0.05$, nObs = 25	0.1131	0.1172	0.1144	0.0017	0.3130	0.0348	0.1165	0.1137	0.0236
$\sigma_u^2 = 0.8$, nObs = 25	0.2403	0.2140	0.2096	0.0000	0.0000	0.4063	0.2177	0.2116	0.2495
$\sigma_u^2 = 0.05$, nObs = 100	0.1006	0.0984	0.0957	0.0617	0.0000	0.0244	0.0988	0.0956	0.0063
$\sigma_u^2 = 0.8$, nObs = 100	0.0956	0.0923	0.0903	0.0155	0.0001	0.0961	0.0931	0.0899	0.0079
$\sigma_u^2 = 0.05$, nObs = 200	0.0860	0.0864	0.0830	0.6693	0.0058	0.0010	0.0868	0.0830	0.0002
$\sigma_u^2 = 0.8$, nObs = 200	0.0793	0.0791	0.0759	0.5887	0.0000	0.0000	0.0792	0.0754	0.0000
P-value	0.0000	0.0000	0.0000				0.0000	0.0000	

Table A4: Mean absolute deviations for scenarios with zero output quantities

	observations with strictly positive output quantities						all observations		
				P-value	P-value	P-value			P-value
all scenarios	0.1375	0.1325	0.1295	0.0000	0.0000	0.0065	0.1340	0.1298	0.0001
share = 0.05	0.1317	0.1315	0.1292	0.9266	0.1807	0.2192	0.1332	0.1300	0.0880
share = 0.1	0.1338	0.1289	0.1289	0.0106	0.0108	0.9780	0.1311	0.1295	0.3690
share = 0.3	0.1471	0.1370	0.1305	0.0000	0.0000	0.0008	0.1377	0.1301	0.0001
P-value	0.0000	0.0000	0.4976				0.0000	0.9040	
$\sigma_u^2 = 0.05$	0.1159	0.1164	0.1136	0.5345	0.0014	0.0001	0.1169	0.1134	0.0000
$\sigma_u^2 = 0.8$	0.1591	0.1486	0.1454	0.0000	0.0000	0.1109	0.1511	0.1462	0.0153
P-value	0.0000	0.0000	0.0000				0.0000	0.0000	
nObs = 25	0.1956	0.1844	0.1811	0.0001	0.0000	0.2111	0.1870	0.1823	0.0790
nObs = 100	0.1176	0.1138	0.1116	0.0000	0.0000	0.0068	0.1152	0.1116	0.0000
nObs = 200	0.0994	0.0992	0.0958	0.7719	0.0000	0.0000	0.0998	0.0955	0.0000
P-value	0.0000	0.0000	0.0000				0.0000	0.0000	
share = 0.05, $\sigma_u^2 = 0.05$	0.1146	0.1170	0.1137	0.0515	0.4678	0.0077	0.1172	0.1138	0.0071
share = 0.1, $\sigma_u^2 = 0.05$	0.1142	0.1157	0.1134	0.2267	0.5212	0.0656	0.1160	0.1134	0.0291
share = 0.3, $\sigma_u^2 = 0.05$	0.1190	0.1165	0.1138	0.0454	0.0000	0.0377	0.1174	0.1130	0.0003
share = 0.05, $\sigma_u^2 = 0.8$	0.1489	0.1461	0.1447	0.4294	0.2304	0.6971	0.1492	0.1461	0.3664
share = 0.1, $\sigma_u^2 = 0.8$	0.1533	0.1422	0.1443	0.0014	0.0107	0.5200	0.1462	0.1455	0.8498
share = 0.3, $\sigma_u^2 = 0.8$	0.1752	0.1576	0.1472	0.0000	0.0000	0.0035	0.1580	0.1471	0.0026
P-value	0.0000	0.0000	0.6310				0.0006	0.7347	
share = 0.05, nObs = 25	0.1846	0.1850	0.1807	0.9290	0.3801	0.3435	0.1872	0.1820	0.2535
share = 0.1, nObs = 25	0.1893	0.1769	0.1796	0.0074	0.0377	0.5410	0.1802	0.1810	0.8576
share = 0.3, nObs = 25	0.2129	0.1914	0.1831	0.0000	0.0000	0.0802	0.1935	0.1840	0.0458
share = 0.05, nObs = 100	0.1139	0.1117	0.1116	0.1329	0.1342	0.9556	0.1134	0.1121	0.3993
share = 0.1, nObs = 100	0.1144	0.1124	0.1115	0.1530	0.0387	0.4957	0.1146	0.1120	0.0455
share = 0.3, nObs = 100	0.1246	0.1173	0.1117	0.0000	0.0000	0.0001	0.1176	0.1107	0.0000
share = 0.05, nObs = 200	0.0967	0.0980	0.0953	0.2167	0.1898	0.0117	0.0990	0.0957	0.0014
share = 0.1, nObs = 200	0.0977	0.0974	0.0954	0.8036	0.0279	0.0491	0.0985	0.0954	0.0018
share = 0.3, nObs = 200	0.1039	0.1023	0.0967	0.1324	0.0000	0.0000	0.1020	0.0955	0.0000
P-value	0.0000	0.0234	0.9155				0.0196	0.8026	
$\sigma_u^2 = 0.05$, nObs = 25	0.1311	0.1352	0.1323	0.0001	0.2529	0.0074	0.1351	0.1318	0.0019
$\sigma_u^2 = 0.8$, nObs = 25	0.2601	0.2336	0.2300	0.0000	0.0000	0.4170	0.2388	0.2328	0.1761
$\sigma_u^2 = 0.05$, nObs = 100	0.1166	0.1138	0.1115	0.0234	0.0000	0.0635	0.1147	0.1114	0.0056
$\sigma_u^2 = 0.8$, nObs = 100	0.1186	0.1138	0.1117	0.0000	0.0000	0.0471	0.1157	0.1118	0.0003
$\sigma_u^2 = 0.05$, nObs = 200	0.1001	0.1001	0.0971	0.9654	0.0053	0.0050	0.1008	0.0970	0.0004
$\sigma_u^2 = 0.8$, nObs = 200	0.0987	0.0984	0.0946	0.5220	0.0000	0.0000	0.0989	0.0941	0.0000
P-value	0.0000	0.0000	0.0000				0.0000	0.0000	

Table A5: Mean rank correlation coefficients for scenarios with zero output quantities

	observations with strictly positive output quantities						all observations		
				P-value	P-value	P-value			P-value
	OD	ODz	SR	OD-ODz	OD-SR	ODz-SR	ODz	SR	ODz-SR
all scenarios	0.5918	0.6098	0.6143	0.0000	0.0000	0.1698	0.6069	0.6163	0.0036
share = 0.05	0.6115	0.6199	0.6203	0.1379	0.1193	0.9463	0.6098	0.6169	0.2037
share = 0.1	0.6002	0.6138	0.6140	0.0190	0.0163	0.9732	0.6025	0.6122	0.0853
share = 0.3	0.5636	0.5956	0.6087	0.0000	0.0000	0.0254	0.6084	0.6197	0.0388
P-value	0.0000	0.0000	0.0003				0.0118	0.0151	
$\sigma_u^2 = 0.05$	0.4216	0.4320	0.4377	0.0007	0.0000	0.0529	0.4327	0.4423	0.0004
$\sigma_u^2 = 0.8$	0.7619	0.7876	0.7910	0.0000	0.0000	0.2109	0.7810	0.7903	0.0006
P-value	0.0000	0.0000	0.0000				0.0000	0.0000	
nObs = 25	0.4612	0.5134	0.5104	0.0000	0.0000	0.6118	0.5008	0.5084	0.1662
nObs = 100	0.6420	0.6488	0.6543	0.1877	0.0163	0.2840	0.6481	0.6574	0.0663
nObs = 200	0.6721	0.6672	0.6782	0.3416	0.2450	0.0356	0.6716	0.6830	0.0279
P-value	0.0000	0.0000	0.0000				0.0000	0.0000	
share = 0.05, $\sigma_u^2 = 0.05$	0.4400	0.4404	0.4448	0.9370	0.3138	0.3471	0.4344	0.4424	0.0879
share = 0.1, $\sigma_u^2 = 0.05$	0.4248	0.4278	0.4358	0.5437	0.0255	0.0967	0.4226	0.4379	0.0014
share = 0.3, $\sigma_u^2 = 0.05$	0.4001	0.4278	0.4325	0.0000	0.0000	0.4124	0.4410	0.4466	0.2427
share = 0.05, $\sigma_u^2 = 0.8$	0.7831	0.7994	0.7958	0.0006	0.0080	0.4255	0.7851	0.7914	0.1834
share = 0.1, $\sigma_u^2 = 0.8$	0.7756	0.7998	0.7921	0.0000	0.0007	0.0875	0.7823	0.7865	0.3691
share = 0.3, $\sigma_u^2 = 0.8$	0.7270	0.7635	0.7850	0.0000	0.0000	0.0000	0.7757	0.7928	0.0002
P-value	0.0002	0.0000	0.6368				0.0000	0.8466	
share = 0.05, nObs = 25	0.4990	0.5204	0.5164	0.0231	0.0635	0.6801	0.5019	0.5089	0.4556
share = 0.1, nObs = 25	0.4818	0.5183	0.5096	0.0002	0.0042	0.3725	0.4901	0.5004	0.2807
share = 0.3, nObs = 25	0.4028	0.5015	0.5053	0.0000	0.0000	0.7312	0.5105	0.5159	0.5690
share = 0.05, nObs = 100	0.6550	0.6608	0.6610	0.5175	0.5022	0.9875	0.6518	0.6589	0.4213
share = 0.1, nObs = 100	0.6434	0.6510	0.6526	0.4032	0.3047	0.8613	0.6455	0.6538	0.3474
share = 0.3, nObs = 100	0.6275	0.6344	0.6494	0.4223	0.0136	0.0897	0.6471	0.6596	0.1478
share = 0.05, nObs = 200	0.6806	0.6785	0.6834	0.8186	0.7520	0.5890	0.6756	0.6829	0.4207
share = 0.1, nObs = 200	0.6755	0.6721	0.6798	0.7122	0.6349	0.4031	0.6719	0.6826	0.2386
share = 0.3, nObs = 200	0.6604	0.6509	0.6714	0.2875	0.2223	0.0223	0.6675	0.6836	0.0660
P-value	0.0000	0.6650	0.9491				0.0017	0.2359	
$\sigma_u^2 = 0.05$, nObs = 25	0.3309	0.3668	0.3665	0.0000	0.0000	0.9620	0.3598	0.3683	0.2012
$\sigma_u^2 = 0.8$, nObs = 25	0.5915	0.6600	0.6544	0.0000	0.0000	0.2832	0.6419	0.6485	0.1597
$\sigma_u^2 = 0.05$, nObs = 100	0.4579	0.4603	0.4667	0.4701	0.0076	0.0481	0.4628	0.4717	0.0026
$\sigma_u^2 = 0.8$, nObs = 100	0.8260	0.8372	0.8419	0.0000	0.0000	0.0060	0.8334	0.8431	0.0000
$\sigma_u^2 = 0.05$, nObs = 200	0.4761	0.4689	0.4799	0.0010	0.0907	0.0000	0.4755	0.4869	0.0000
$\sigma_u^2 = 0.8$, nObs = 200	0.8681	0.8655	0.8766	0.0108	0.0000	0.0000	0.8678	0.8792	0.0000
P-value	0.0000	0.0000	0.0000				0.0000	0.0000	

Table A6: Mean biases for scenarios with zero output quantities

	observations with strictly positive output quantities						all observations		
				P-value	P-value	P-value			P-value
	OD	ODz	SR	OD-ODz	OD-SR	ODz-SR	ODz	SR	ODz-SR
all scenarios	0.0507	0.0368	0.0394	0.0000	0.0000	0.1257	0.0370	0.0410	0.0197
share = 0.05	0.0420	0.0424	0.0393	0.9114	0.3559	0.3072	0.0441	0.0404	0.2200
share = 0.1	0.0436	0.0346	0.0373	0.0025	0.0332	0.3514	0.0367	0.0389	0.4616
share = 0.3	0.0664	0.0334	0.0417	0.0000	0.0000	0.0073	0.0302	0.0437	0.0000
P-value	0.0000	0.0012	0.2247				0.0000	0.1418	
$\sigma_u^2 = 0.05$	0.0342	0.0294	0.0300	0.0184	0.0330	0.7829	0.0290	0.0303	0.5075
$\sigma_u^2 = 0.8$	0.0672	0.0442	0.0489	0.0000	0.0000	0.0897	0.0450	0.0517	0.0161
P-value	0.0000	0.0000	0.0000				0.0000	0.0000	
nObs = 25	0.1284	0.1019	0.1016	0.0000	0.0000	0.9389	0.1057	0.1036	0.5997
nObs = 100	0.0151	0.0059	0.0100	0.0000	0.0159	0.0557	0.0054	0.0116	0.0028
nObs = 200	0.0086	0.0025	0.0067	0.0003	0.2499	0.0097	-0.0001	0.0078	0.0000
P-value	0.0000	0.0000	0.0000				0.0000	0.0000	
share = 0.05, $\sigma_u^2 = 0.05$	0.0296	0.0356	0.0296	0.0875	0.9974	0.0871	0.0367	0.0304	0.0698
share = 0.1, $\sigma_u^2 = 0.05$	0.0283	0.0302	0.0273	0.5760	0.7783	0.4026	0.0313	0.0282	0.3585
share = 0.3, $\sigma_u^2 = 0.05$	0.0447	0.0224	0.0330	0.0000	0.0007	0.0028	0.0189	0.0324	0.0001
share = 0.05, $\sigma_u^2 = 0.8$	0.0545	0.0491	0.0490	0.2675	0.2500	0.9838	0.0514	0.0504	0.8374
share = 0.1, $\sigma_u^2 = 0.8$	0.0590	0.0390	0.0473	0.0000	0.0152	0.0733	0.0421	0.0496	0.1089
share = 0.3, $\sigma_u^2 = 0.8$	0.0882	0.0445	0.0504	0.0000	0.0000	0.2347	0.0414	0.0551	0.0068
P-value	0.0009	0.0385	0.8757				0.0723	0.8555	
share = 0.05, nObs = 25	0.1076	0.1063	0.1008	0.8415	0.3086	0.4251	0.1103	0.1025	0.2469
share = 0.1, nObs = 25	0.1150	0.0921	0.0965	0.0008	0.0067	0.5053	0.0987	0.0983	0.9604
share = 0.3, nObs = 25	0.1627	0.1075	0.1076	0.0000	0.0000	0.9935	0.1081	0.1100	0.7823
share = 0.05, nObs = 100	0.0111	0.0113	0.0097	0.9602	0.7044	0.6682	0.0128	0.0108	0.5903
share = 0.1, nObs = 100	0.0097	0.0075	0.0089	0.5306	0.8056	0.6982	0.0086	0.0105	0.5857
share = 0.3, nObs = 100	0.0244	-0.0009	0.0113	0.0000	0.0005	0.0007	-0.0052	0.0135	0.0000
share = 0.05, nObs = 200	0.0074	0.0095	0.0074	0.4550	0.9880	0.4608	0.0091	0.0079	0.6886
share = 0.1, nObs = 200	0.0062	0.0043	0.0065	0.4996	0.9031	0.4206	0.0029	0.0077	0.0895
share = 0.3, nObs = 200	0.0122	-0.0063	0.0062	0.0000	0.0398	0.0000	-0.0124	0.0077	0.0000
P-value	0.0000	0.0004	0.4391				0.0004	0.4056	
$\sigma_u^2 = 0.05$, nObs = 25	0.0550	0.0444	0.0453	0.0050	0.0084	0.8205	0.0456	0.0459	0.9289
$\sigma_u^2 = 0.8$, nObs = 25	0.2019	0.1595	0.1580	0.0000	0.0000	0.8117	0.1658	0.1613	0.4688
$\sigma_u^2 = 0.05$, nObs = 100	0.0231	0.0236	0.0230	0.8882	0.9793	0.8645	0.0225	0.0233	0.8308
$\sigma_u^2 = 0.8$, nObs = 100	0.0071	-0.0117	-0.0030	0.0000	0.0000	0.0000	-0.0117	-0.0000	0.0000
$\sigma_u^2 = 0.05$, nObs = 200	0.0245	0.0202	0.0216	0.1446	0.3083	0.6298	0.0188	0.0217	0.3217
$\sigma_u^2 = 0.8$, nObs = 200	-0.0074	-0.0152	-0.0082	0.0000	0.5262	0.0000	-0.0191	-0.0062	0.0000
P-value	0.0000	0.0000	0.0000				0.0000	0.0000	